

Interpolation based Fuzzy Automaton for Human-Robot Interaction

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Abstract: One way of handling Human-Robot Interaction (HRI) is based on the concept, that the robot acts like an animal companion to human. According to this paradigm the Robot should not be molded to mimic the human being, and form human-to-human like communication, but to follow the existing biological examples and form inter-species interaction. The 20.000 year old human-dog relationship is a good example for this paradigm of the HRI, as interaction of different species. One good reason of this approach in HRI is the lack of the “uncanny valley” effect i.e. increasing similarity of robots to humans will actually increase the chances that humans refuse interaction (will be frightened). In this paper, for ethologically inspired HRI model implementation, a fuzzy model structure built upon the framework of low computational demand Fuzzy Rule Interpolation (FRI) methods and fuzzy automaton is suggested. The application of FRI methods fits well the conceptually “sparse rule-based” structure of the existing descriptive verbal ethological models. (In case of the descriptive verbal ethological models, the “completeness” of the rule-base is not required). The main benefit of the FRI method adaptation in ethological model implementation is the fact, that it has a simple rule-based knowledge representation format. Because of this, even after numerical optimization of the model, the rules are still “human readable”, and helps the formal validation of the model by the ethological experts. On the other side due to the FRI base, the model has still low computational demand and fits directly the requirements of the embedded implementations. For demonstrating the applicability of the proposed structure, some components of a human-dog interaction FRI model, which also suitable for HRI, will be briefly introduced in this paper.

Keywords: Fuzzy Rule Interpolation, Fuzzy Automaton, Behaviour-based Control, Human-Robot Interaction.

1. INTRODUCTION

In recent years there has been an increased interest in the development of Human-Robot Interaction (HRI). Researchers have assumed that HRI could be enhanced if these intelligent systems were able to express some pattern of sociocognitive and socioemotional behaviour (e.g. Dautenhahn 2007). Such approach needed an interaction among various scientific disciplines including psychology, cognitive science, social sciences, artificial intelligence, computer science and robotics. The main goal has been to find ways in which humans can interact with these systems in a “natural” way. Recently HRI has become very user oriented, that is, the performance of the robot is evaluated from the user’s perspective. This view also reinforces arguments that robots do not only need to display certain emotional and cognitive skills but also showing features of individuality. Generally however, most socially interactive robots are not able to

support long-term interaction with humans, and the interest shown toward them wears out rapidly.

2. Challenges in HRI

The design of socially interactive robots has faced many challenges. Despite major advances there are still many obstacles to be solved in order to achieve a natural-like interaction between robots and humans.

The “uncanny valley” effect: Mori (Mori, 1970) assumed that the increasing similarity of robots to humans will actually increase the chances that humans refuse interaction (will be frightened from) very human-like agents. Although many take this effect for granted only little actual research was devoted to this issue. Many argue that once an agent passes certain level of similarity, as it is the case in the most recent visual characters in computer graphics, people will treat them just as people (Potal, 2008). However, in the case of 3D robots, the answer is presently less clear, as up do date

technology is very crude in reproducing natural-like behaviour, emotions and verbal interaction. Thus for robotics the uncanny valley effect will present a continuing challenge in the near future.

In spite of the huge advances in robotics current socially interactive systems fail both with regard to motor and cognitive capacities, and in most cases can interact only in a very limited way with the human partner. We see this as a major discrepancy that is not easy to solve because there is a big gap between presently available technologies (hardware and software) and the desire for achieving human-like cognitive and motor capacities. As a consequence recent socially interactive robots have only a restricted appeal to humans, and after losing the effect of novelty the interactions break down rapidly.

The planning and construction of biologically or psychologically inspired robots depends crucially of the current understanding of human motor and mental processes. However, these are one of the most complex phenomena of life! Thus it is certainly possible that human mental models of abilities like “intention”, “human memory” etc., which serve at present as the underlying concepts for control socially interactive robots, will be proved to be faulty.

Because of the goal of mimicking a human, socially interactive robots do not utilise more general human abilities that have evolved as general skills for social interaction. Further, the lack of evolutionary approach in conceptualizing the design of such robots hinders further development, and reinforces that the only goal in robotics should be the produce “as human-like as possible” agents.

3. Ethologically Inspired HRI Model

In order to overcome some of the challenges presented above ethologically inspired HRI models can be applied. The concept of ethologically inspired HRI models allows the study of individual interactions between animals and animals and humans. If one defines robots as mechanical or electronic agents that extend human capacities then the dog (which has been domesticated by humans) represent the first biological robot because some time after domestication dogs were utilized as an aid in hunting, animal husbandry, warfare, protection, transport etc (Miklósi, 2007). The long-term (since cc 20.000 years) and successful human-dog interaction shows that humans have the ability to develop social interaction with very different agents. The human-dog relationship rests critically on our ability to produce and understand various forms of communicative cues that are emitted in an inter-specific relationship in which the two members’ signalling behaviour overlaps only to certain extent. Human behaviour evolution has selected for increased ability to form social contact with any creatures which originates in the very social nature of nursing (parental) behaviour in humans which is unique in the Primates. Humans also show a preference to use social relationship for joint action in cooperative settings. Finally, humans have the mental capacities (and the preference) to attribute certain human-like mental capacities to other agents (even to non-

living things) which also facilitates the interaction between them.

Such robots do not have to rely on the exact copy of human social behaviour (including language etc) but should be able to produce social behaviours that provide a minimal set of actions on which human-robot cooperation can be achieved. Such basic models of robots could be “improved” with time making the HRI interaction more complex.

4. FRI in HRI Model

In ethological modeling, mass of expert knowledge exists in the form of expert’s rules. Most of them are descriptive verbal ethological models. The knowledge representation of verbal expert’s rules can be very simply translated to the structure of fuzzy rules, transforming the initially verbal ethological models to a fuzzy model.

On the other hand, in case of the descriptive verbal ethological models, the “completeness” of the rule-base is not required (thanks to the descriptive manner, of the model), which makes implementation difficulties in classical fuzzy rule based systems, and classical fuzzy reasoning methods (e.g. the Zadeh-Mamdani-Larsen Compositional Rule of Inference (CRI) (Zadeh, 1973) (Mamdani, 1975) (Larsen, 1980) or the Takagi - Sugeno fuzzy inference (Sugeno, 1985) (Takagi and Sugeno, 1985)). Another problem of the complete rule base is the space complexity. The size of a complete rule base grows exponentially with the number of the rule antecedent dimensions. A model having more than 7-8 input dimensions is practically unimplementable as a complete rule base. On the other hand, in the descriptive verbal ethological models the 10-20 input variables are common. Classical fuzzy reasoning methods are assuming the completeness of the fuzzy rule base. If there are some rules missing i.e. the rule base is “sparse”, observations may exist which hit no rule in the rule base and therefore no conclusion can be obtained. One way of handling the “fuzzy dot” knowledge representation in case of sparse fuzzy rule bases is the application of the Fuzzy Rule Interpolation (FRI) methods, where the derivable rules are deliberately missing. Since FRI methods can provide reasonable (interpolated) conclusions even if none of the existing rules fires under the current observation. From the beginning of 1990s numerous FRI methods have been proposed (Wong, et. al., 2006).

5. The “FIVE” FRI

An application oriented aspect of the fuzzy rule interpolation emerges in the concept of “FIVE”. The fuzzy reasoning method “FIVE” (Fuzzy Interpolation based on Vague Environment, originally introduced in (Kovács, 1996), (Kovács and Kóczy 1997a) and (Kovács and Kóczy 1997b)) was developed to fit the speed requirements of direct fuzzy control, where the conclusions of the fuzzy controller are applied directly as control actions in a real-time system.

The main idea of the FIVE is based on the fact that most of the control applications serves crisp observations and requires crisp conclusions from the controller. Adopting the idea of the vague environment (VE) (Klawonn, 1994), FIVE can

handle the antecedent and consequent fuzzy partitions of the fuzzy rule base by scaling functions (Kovács and Kóczy 1997b) and therefore turn the fuzzy interpolation to crisp interpolation.

The idea of a VE is based on the similarity (in other words: indistinguishability) of the considered elements. In VE the fuzzy membership function $\mu_A(x)$ is indicating level of similarity of x to a specific element a that is a representative or prototypical element of the fuzzy set $\mu_A(x)$, or, equivalently, as the degree to which x is indistinguishable from a (Kovács and Kóczy 1997b). Therefore the α -cuts of the fuzzy set $\mu_A(x)$ are the sets which contain the elements that are $(1-\alpha)$ -indistinguishable from a . Two values in a VE are ε -distinguishable if their distance is greater than ε . The distances in a VE are weighted distances. The weighting factor or function is called *scaling function (factor)* (Kovács and Kóczy 1997b). If VE of a fuzzy partition (the scaling function or at least the approximate scaling function (Kovács, 1996), (Kovács and Kóczy 1997b)) exists, the member sets of the fuzzy partition can be characterized by points in that VE (see e.g. scaling function s on fig. 1). Therefore any crisp interpolation, extrapolation, or regression method can be adapted very simply for FRI (Kovács, 1996), (Kovács and Kóczy 1997b). Because of its simple multidimensional applicability, in FIVE the *Shepard operator* based interpolation (first introduced in (Shepard, 1968)) is adapted (see e.g. fig. 1).

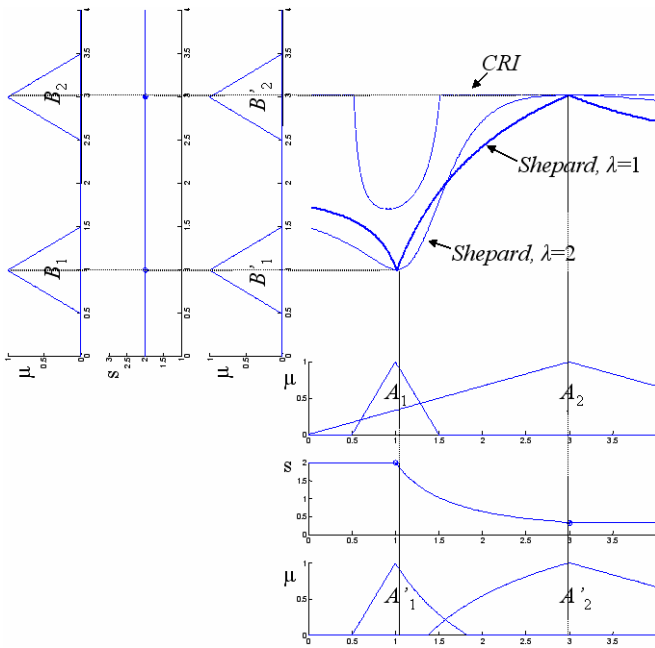


Fig.1: Interpolation of two fuzzy rules ($R_i: A_i \rightarrow B_i$), by the Shepard operator based FIVE, and for comparison the min-max CRI with COG defuzzification.

In this case if the fuzzy rules R_k has the following form:

If $x_1=A_{k,1}$ **And** $x_2=A_{k,2}$ **And** ... **And** $x_m=A_{k,m}$ **Then** $y=B_k$

The FIVE interpolation can be expressed by the following formula:

$$\delta_s(b_0, y(\mathbf{x})) = \begin{cases} \delta_s(b_0, b_k) & \text{if } \mathbf{x} = \mathbf{a}_k \text{ for some } k, \\ \left(\frac{\sum_{k=1}^r \delta_s(b_0, b_k) / \delta_{s,k}^\lambda}{\sum_{k=1}^r 1 / \delta_{s,k}^\lambda} \right) & \text{otherwise.} \end{cases}$$

where $y(\mathbf{x})$ is the requested one dimensional conclusion, r is the number of the fuzzy rules in the rule base R , $\lambda > 0$ is a parameter of the Shepard operator, b_0 is the first element of the one dimensional consequence universe ($Y: b_0 \leq y, \forall y \in Y$), and $\delta_{s,k}, \delta_s$ can be calculated by the following formula:

$$\delta_{s,k} = \delta_s(\mathbf{a}_k, \mathbf{x}) = \left[\sum_{i=1}^m \left(\int_{a_{k,i}}^{x_i} s_{X_i}(x_i) dx_i \right)^2 \right]^{1/2},$$

$$\delta_s(b_0, b_k) = \int_{b_0}^{b_k} s_Y(y) dy,$$

where s_{X_i} is the i^{th} scaling function of the m dimensional antecedent universe, \mathbf{x} is the m dimensional crisp observation, \mathbf{a}_k are the cores of the m dimensional fuzzy rule antecedents A_k and s_Y is the i^{th} scaling function of the one dimensional consequent universe, b_k are the cores of the one dimensional fuzzy rule consequents B_k .

An implementation of FIVE as a component of the FRI Matlab Toolbox (Johanyák, et.al., 2006) can be downloaded from (FRI Toolbox).

6. FRI based Fuzzy Automaton for HRI

For implementing ethologically inspired HRI models, in this paper the classical behaviour-based control structure is suggested. In behaviour-based control systems (a good overview can be found in (Pirjanian, P., 1999)), the actual behaviour of the system is formed as one of the existing behaviours (which fits best the actual situation), or a kind of fusion of the known behaviours appeared to be the most appropriate to handle the actual situation. This structure has two main tasks. The first is a decision, which behaviour is needed in an actual situation, and the levels of their necessities in case of behaviour fusion. The second is the way of the behaviour fusion. The first task can be viewed as an actual system state approximation, where the actual system state is the set of the necessities of the known behaviours needed for handling the actual situation. The second is the fusion of the known behaviours based on these necessities.

In case of the suggested fuzzy behaviour based control structures both tasks are solved by FRI systems. If the behaviours are also implemented on FRI models, the behaviours together with the behaviour fusion modules form a hierarchical FRI system.

The application of FRI methods in direct fuzzy logic control systems gives a simplified way for constructing the fuzzy rule base. The rule base of a fuzzy interpolation-based model, is not necessarily complete, it could contain the most significant fuzzy rules only without risking the chance of having no conclusion for some of the observations. In other words,

during the construction of the fuzzy model, it is enough to concentrate on the main actions (the rules which could be deduced from the others could be intentionally left out from the model).

7. The Suggested FRI Behaviour-based Structure

In case of pure FRI based fuzzy behaviour-based control structures all the main tasks of the behaviour-based control are implemented on FRI models. Such a structure is introduced on Fig.2. The three main tasks, the behaviour coordination, the behaviour fusion, and the behaviours themselves are FRI models.

For demonstrating the main benefits of the FRI model in behaviour-based control, in this paper we concentrate only on the (usually) most heuristic part of the structure, on the behaviour coordination. The task of behaviour coordination is to determine the necessities of the known behaviours needed for handling the actual situation. In the suggested behaviour-based control structure, for this task the finite state fuzzy automaton is adapted (Fig.2.) (Kovács, 2000), where the state of the finite state fuzzy automaton is the set of the suitabilities of the component behaviours. This solution is based on the heuristic, that the necessities of the known behaviours for handling a given situation can be approximated by their suitability. And the suitability of a given behaviour in an actual situation can be approximated by the similarity of the situation and the prerequisites of the behaviour. (Where the prerequisites of the behaviour is the description of the situations where the behaviour is applicable). In this case instead of determining the necessities of the known behaviours, the similarities of the actual situation to the prerequisites of all the known behaviours can be approximated.

Thus the first step of the system state approximation is determining the similarities of the actual situation to the prerequisites of all the known behaviours – applying the terminology of fault classification, it is the symptom evaluation (see on Fig.2.). The task of symptom evaluation is basically a series of similarity checking between an actual symptom (observations of the actual situation) and a series of known symptoms (the prerequisites – symptom patterns – of the behaviour components). These symptom patterns are characterising the systems states where the corresponding behaviours are valid. Based on these patterns, the evaluation of the actual symptom is done by calculating the similarity values of the actual symptom (representing the actual situation) to all the known symptoms patterns (the prerequisites of the known behaviours). There are many methods for fuzzy logic symptom evaluation. For example fuzzy classification methods e.g. the Fuzzy c-Means fuzzy clustering algorithm (Bezdek, 1981) can be adopted, where the known symptoms patterns are the cluster centres, and the similarities of the actual symptom to them can be fetched from the fuzzy partition matrix. On the other hand, having a simple situation, the fuzzy logic symptom evaluation could be an FRI model too.

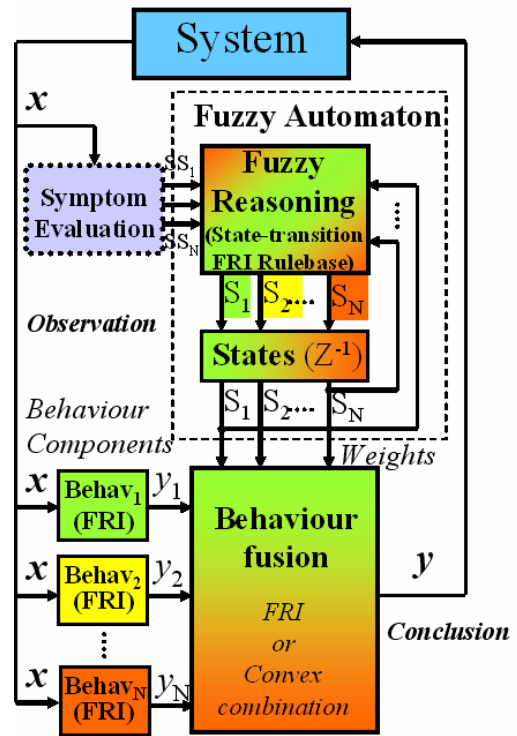


Fig.2. The suggested FRI behaviour-based structure.

One of the main difficulties of the system state approximation is the fact, that most cases the symptoms of the prerequisites of the known behaviours are strongly dependent on the actual behaviour of the system. Each behaviour has its own symptom structure. In other words for the proper system state approximation, the approximated system state is needed itself. A very simple way of solving this difficulty is the adaptation of fuzzy automaton. This case the state vector of the automaton is the approximated system state, and the state-transitions are driven by fuzzy reasoning (Fuzzy state-transition rule base on Fig.2.), as a decision based on the previous actual state (the previous iteration step of the approximation) and the results of the symptom evaluation.

For demonstrating the simplicity of defining the rule base for the FRI model, a small example will be introduced in the followings.

8. Example

The example is a tiny fragment of a more complex ethological model of an RDog behaving in an unfamiliar room in interaction with its owner and an unknown human ("Owner" and "Human2" on Fig.3). In the name of "Rdog" the "R" stands for "Robot" i.e. the dog in question is a Robot. The "real" version of the "Rdog" is introduced on Fig.5.

The example behaviour is built upon two component behaviours, namely "RdogExploresTheRoom" and "RdogGoesToDoor" built separately.

The "RdogExploresTheRoom" is an exploration dog activity, in which the dog "looks around" in an unknown environment (see the "ellipsoid" track on Fig.3). The

“RDOGgoesToDoor” is a simple dog activity, in which the dog goes to the door, and than stands (sits) in front of it.

The example is the definition of the related state-transition FRI rules of the fuzzy automaton acts as behaviour coordination.

The states concerned in the example are the following:

“Missing the owner mood of the RDOG” (RDOGmissTheOwner) and “Anxiety level of the RDOG” (RDOGAnxietyLevel): “hidden” states, which have no direct task in controlling any of the above mentioned behaviours, but has an importance in the state-transition rule base.

“Going to the door mood of the RDOG” (RDOGgoesToDoor) and “Room exploration mood of the RDOG” (RDOGexploresTheRoom): states, which have also direct task in controlling the corresponding “RDOGexploresTheRoom” and “RDOGgoesToDoor” behaviours.

As a possible rule base structure for the state-transitions of the fuzzy automaton, the following is defined (a tiny fragment of a more complex rule base):

State-transition rules related to the missing the owner mood (state) of the RDOG:

If OwnerInTheRoom=*False* **Then**
RDOGmissTheOwner=*Increasing*

If OwnerInTheRoom=*True* **Then**
RDOGmissTheOwner=*Decreasing*

State-transition rules related to the anxiety level (state) of the RDOG:

If OwnerToDogDistance=*Small* **And**
Human2ToDogDistance=*High* **Then**
RDOGAnxietyLevel=*Decreasing*

If OwnerToDogDistance=*High* **And**
Human2ToDogDistance=*Small* **Then**
RDOGAnxietyLevel=*Increasing*

State-transition rules related to the going to the door mood (state) of the RDOG:

If OwnerInTheRoom=*False* **And**
RDOGmissTheOwner=*High* **Then**
RDOGgoesToDoor=*High*

If OwnerInTheRoom=*True* **Then**
RDOGgoesToDoor=*Low*

State-transition rules related to the room exploration mood (state) of the RDOG:

If RDOGAnxietyLevel=*Low* **And**
OwnerStartsGame=*False* **And**
ThePlaceIsUnknown=*High* **Then**
RDOGexploresTheRoom=*High*

If ThePlaceIsUnknown=*Low* **Then**
RDOGexploresTheRoom=*Low*

If RDOGAnxietyLevel=*High* **Then**
RDOGexploresTheRoom=*Low*

where the text in *Italic* are the linguistic terms (fuzzy sets) of the FRI rule base.

Please note that the rule base is sparse. It contains the main state-transition FRI rules only.

A sample run of the example is introduced on Fig.3 and Fig.4. At the beginning of the scene, the owner is in the room and the Human2 is outside. The place is unknown for the dog (“ThePlaceIsUnknown=*High*” in the rule base). according to the above rule base, the dog starts to explore the room. At around the step count 17, the owner of the dog left the room, than “Human2” enters and stay inside. As an effect of the changes (according to the above state-transition rule base), the anxiety level of the dog and the “missing the owner” is increasing, and as a result, the dog goes and stays at the door, where the owner has left the room. See example run tracks on Fig.3 and state changing on Fig.4.

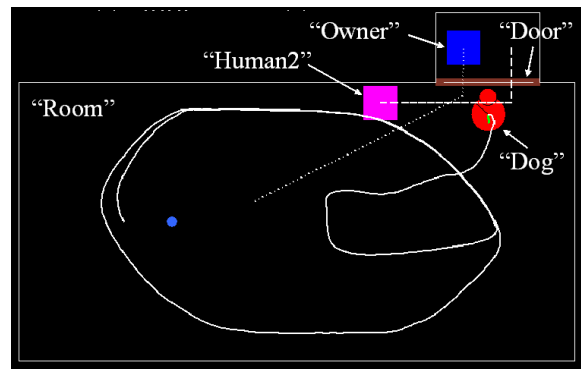


Fig.3. Tracks of a sample run. Continuous line for the for the dog, dotted for the owner and dashed for the Human2.

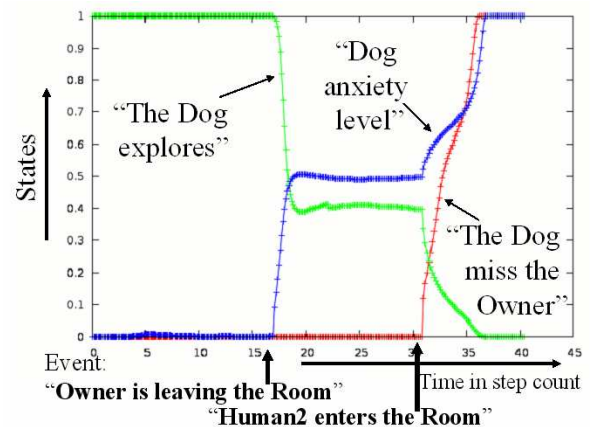


Fig. 4. Some state changes during the sample run introduced on Fig.3.

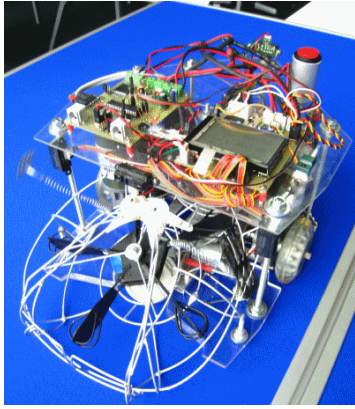


Fig. 5. The “real” RDog.

9. CONCLUSIONS

The goal of this paper was to suggest a behaviour-based structure built from Fuzzy Rule Interpolation (FRI) models and FRI automaton for handling Human-Robot Interaction (HRI) placed on ethological model basis. The suggested structure is simple and could be implemented to be quick enough to fit the requirements of direct real-time HRI applications. It is an easily built and simply adaptable structure for many application areas (see e.g. (Kovács, Sz., 2002) as an application area in user adaptive emotional and information retrieval systems). The implementation of FRI reasoning methods in HRI applications simplifies the task of fuzzy rule base creation. The FRI rule base is not needed to be complete, so it is enough to concentrate on the main control actions, or even the rules can be added simply piece by piece.

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REFERENCES

- J.C. Bezdek (1981). *Pattern Recognition with Fuzzy Objective Function*, Plenum Press, New York.
- K. Dautenhahn (2007). Methodology and themes of human-robot interaction: A growing research field. *Int. J. of Advanced Robotic Systems*, 4, 103- 108.
- Z. C. Johanyák, D. Tikk, S. Kovács and K.K. Wong (2006). Fuzzy Rule Interpolation Matlab Toolbox - FRI Toolbox, Proc. of the IEEE World Congress on Computational Intelligence (WCCI'06), 15th Int. Conf. on Fuzzy Systems (FUZZ-IEEE'06), July 16-21, Vancouver, BC, Canada, pp. 1427-1433.
- F. Klawonn (1994). Fuzzy Sets and Vague Environments, *Fuzzy Sets and Systems*, 66, pp.207-221.
- Sz. Kovács (1996). New Aspects of Interpolative Reasoning, Proceedings of the 6th. International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems, Granada, Spain, pp. 477-482.
- Sz. Kovács and L.T. Kóczy (1997a). Approximate Fuzzy Reasoning Based on Interpolation in the Vague Environment of the Fuzzy Rule base as a Practical Alternative of the Classical CRI, Proceedings of the 7th International Fuzzy Systems Association World Congress, Prague, Czech Republic, 144-149.
- Sz. Kovács and L.T. Kóczy (1997b). The use of the concept of vague environment in approximate fuzzy reasoning, *Fuzzy Set Theory and Applications*, Tatra Mountains Mathematical Publications, Mathematical Institute Slovak Academy of Sciences, Bratislava, Slovak Republic, vol.12, pp. 169-181.
- Sz. Kovács (2000). “Interpolative Fuzzy Reasoning and Fuzzy Automaton in Adaptive System Applications”, Proceedings of the IIZUKA2000, 6th International Conference on Soft Computing, October 1-4, Iizuka, Fukuoka, Japan, pp.777-784.
- Sz. Kovács (2002). “Fuzzy Reasoning and Fuzzy Automata in User Adaptive Emotional and Information Retrieval Systems”, Proceedings of the 2002 IEEE International Conference on Systems, Man and Cybernetics, October 6-9, Hammamet, Tunisia, 02CH37349C, ISBN: 0-7803-7438-X, WP1N5, p.6.
- Sz. Kovács (2005). Interpolative Fuzzy Reasoning in Behaviour-based Control, *Advances in Soft Computing*, Vol. 2, Computational Intelligence, Theory and Applications, Bernd Reusch (Ed.), Springer, Germany, ISBN 3-540-22807-1, pp.159-170.
- P. M. Larsen (1980). Industrial application of fuzzy logic control. *Int. J. of Man Machine Studies*, (12) 4, 3-10.
- E. H. Mamdani and S. Assilian (1975). An experiment in linguistic synthesis with a fuzzy logic controller. *Int. J. of Man Machine Studies*, (7), 1-13.
- Á. Miklósi (2007). *Dog behaviour, evolution and cognition*. Oxford University Press.
- Masahiro Mori (1970). The Uncanny Valley, *Energy*, 7 (4), pp. 33-35.
- P. Pirjanian (1999). “Behavior Coordination Mechanisms - State-of-the-art”, Tech-report IRIS-99-375, Institute for Robotics and Intelligent Systems, School of Engineering, University of Southern California, <http://www-robotics.usc.edu/~paolo/publications/bcm.ps.gz>, Oct.
- M. Potal (2008). Overcoming the uncanny valley. *IEEE Computer Graphics and Applications*, 4 11-17.
- D. Shepard (1968). A two dimensional interpolation function for irregularly spaced data, Proc. 23rd ACM Internat. Conf., pp. 517-524.
- M. Sugeno (1985). An introductory survey of fuzzy control. *Information Science*, (36), 59-83.
- T. Takagi and M. Sugeno (1985). Fuzzy identification of systems and its applications to modeling and control. *IEEE Trans. on SMC*, (15), 116-132.
- K W Wong, D Tikk, T D Gedeon and L T Kóczy (2006). Fuzzy rule Interpolation for Multidimensional Input Spaces With Applications: A Case Study, *IEEE T FUZZY SYST* 13: (6) pp.809-819.
- L. A. Zadeh (1973). Outline of a new approach to the analysis of complex systems and decision processes. *IEEE Trans. on SMC*, (3), 28-44.

The FRI Toolbox is available at: <http://fri.gamf.hu>